

The 39th Annual AAAI Conference on Artificial Intelligence

CPML Bridge Program

Integrating Automated Reasoning and Machine Learning for Structured Prediction

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On the job

market!



Machine Learning

Automated Reasoning



Machine Learning

Bottom-up and Inductive: Fit data distributions well.

- E.g.,
 - Perceptron
 - Support vector machine
 - Generative model







Automated Reasoning



Machine Learning

Bottom-up and Inductive: Fit data distributions well.

- E.g.,
 - Perceptron
 - Support vector machine
 - Generative model







Automated Reasoning

- Top-down and deductive: precise models from problem description.
- E.g.,
 - SATisfiability (SAT) solvers
 - Satisfiability Module Theory (SMT) solver
 - Mixed Integer Programming (MIP) solver





Machine Learning

- Challenging in providing formal guarantees.
- Hallucination: generated outputs are false or fabricated.
- May violate constraints in rare and unseen situations.

Automated Reasoning



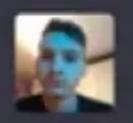
Machine Learning

- Challenging in providing formal guarantees.
- Hallucination: generated outputs are false
 Difficult to adapt to evolving data or fabricated.
- May violate constraints in rare and unseen
 Cannot understand data like text and images.

Automated Reasoning

• Rigid models: problem formulation must be agreed a-priori.

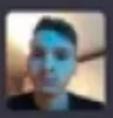




Mike's mum had 4 kids; 3 of them are Luis, Drake and Matilda. What is the name of 4th kid?



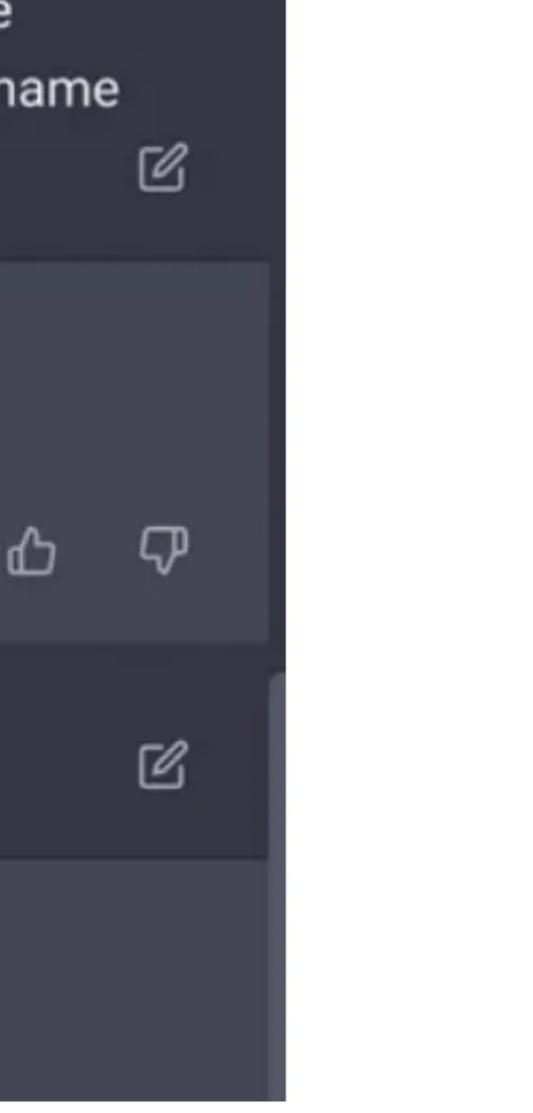
It is not possible to determine the name of the fourth child without more information.

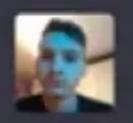


The answer is in the question



I apologize for the confusion. It is not possible to determine the name

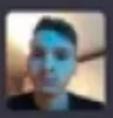




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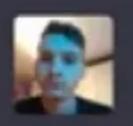
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ChatGPT struggle with questions in logical reasoning and context comprehension.



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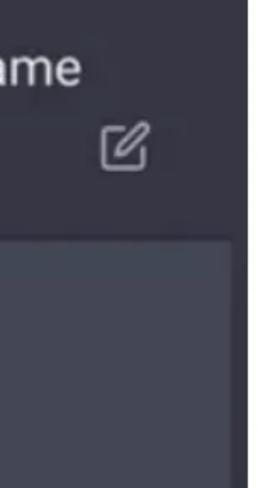
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FINANCIAL TIMES

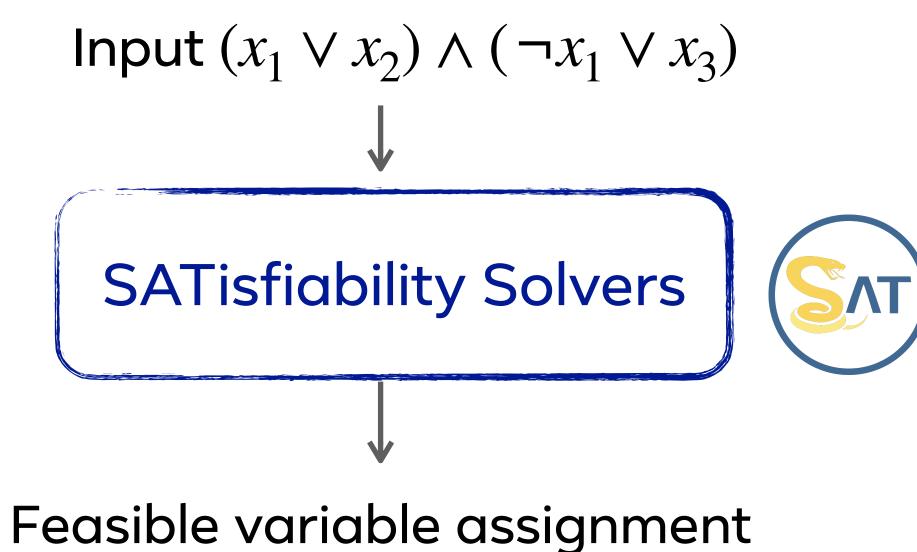
Yann LeCun, chief AI scientist at the social media giant that owns Facebook and Instagram, said LLMs had "very limited understanding of logic . . . do not understand the physical world, do not have persistent memory, cannot reason in any reasonable definition of the term and cannot plan . . . hierarchically".

not possible to determine the name



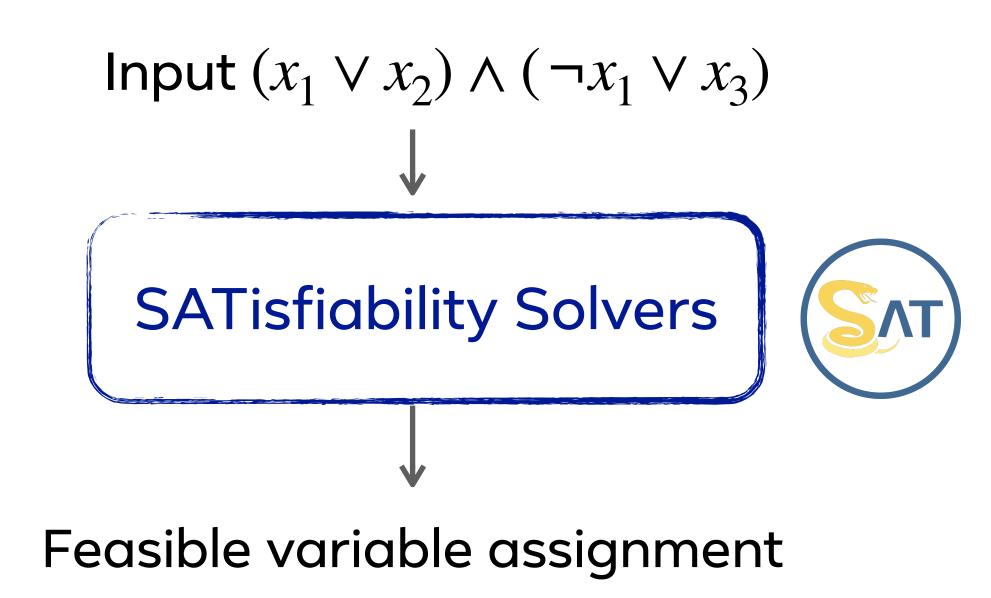
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Automated Reasoning has intrinsic difficulties



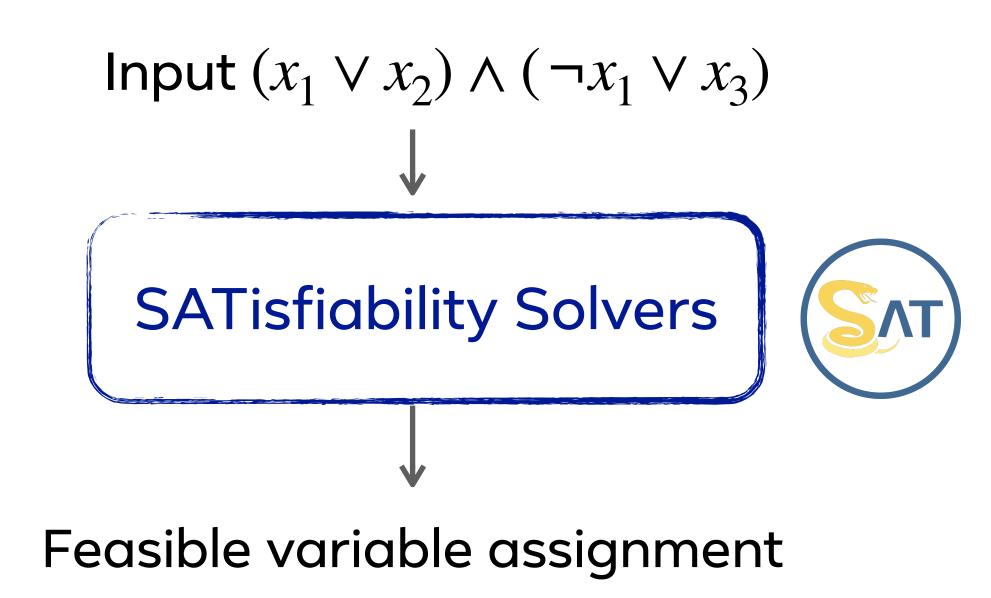


Automated Reasoning has intrinsic difficulties



hard to encode data distribution.

Automated Reasoning has intrinsic difficulties



- hard to encode data distribution.
- hard to handle complex input data, like language and image

Machine Learning

Automated Reasoning



Machine Learning



Learn data distribution

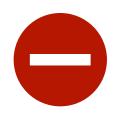
Automated Reasoning



Machine Learning



Learn data distribution



Provide formal guarantees

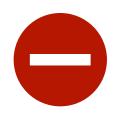
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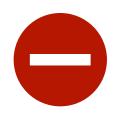
Feasible output



Machine Learning



Learn data distribution



Provide formal guarantees

Automated Reasoning

Feasible output

Encode evolving data distribution



Machine Learning



Learn data distribution

Provide formal guarantees

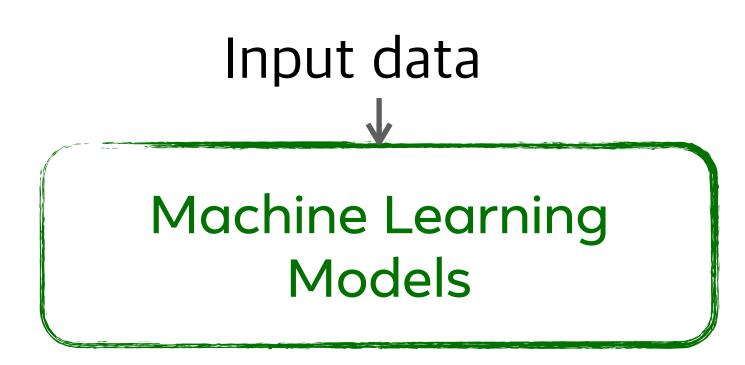
A class of Structured prediction problems are beyond the reach of machine learning and automated reasoning, when they are applied in isolation.

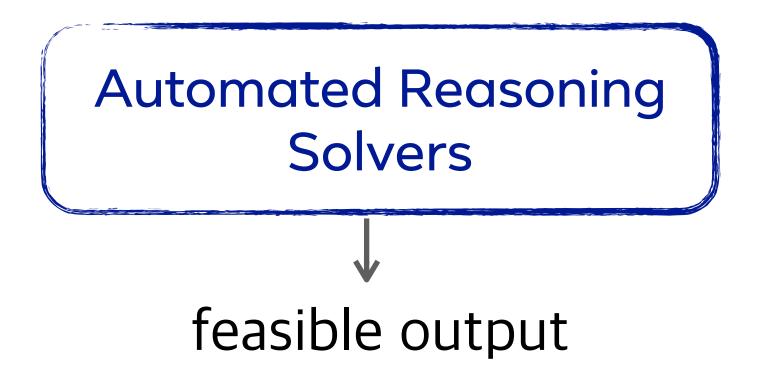
Automated Reasoning

Feasible output

Encode evolving data distribution

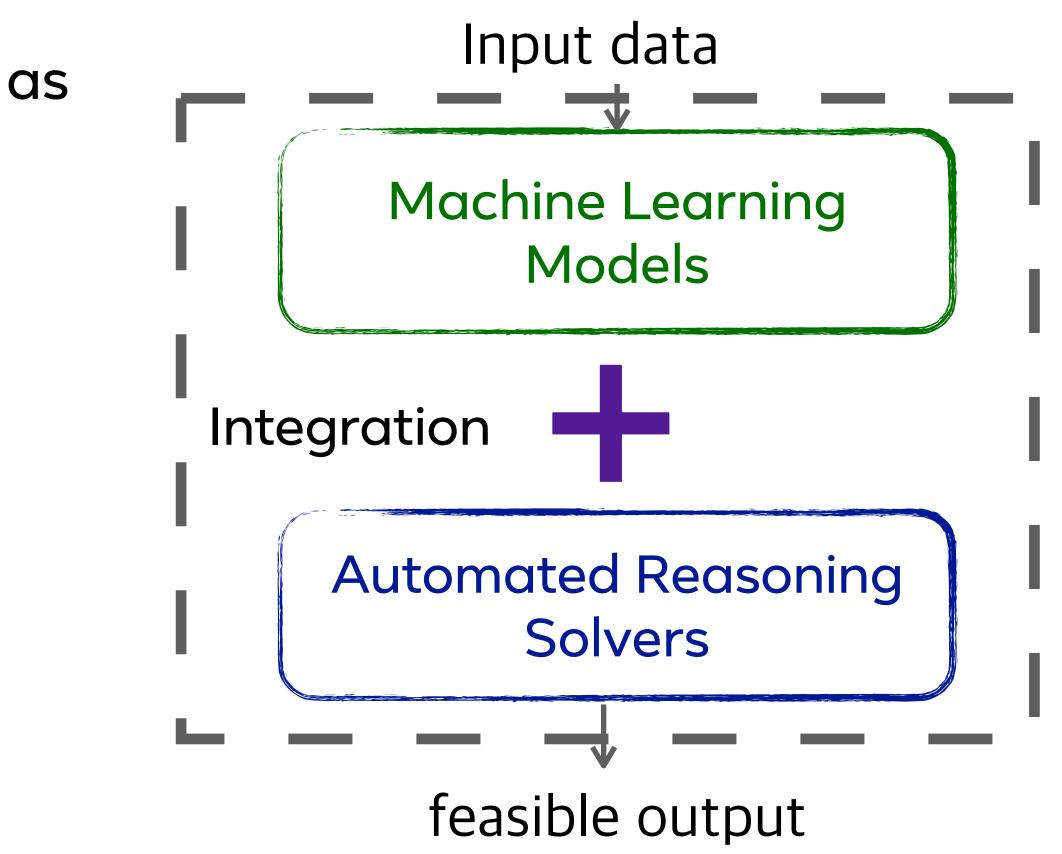








Key insight: Embed reasoning solvers as differentiable modules into neural networks.

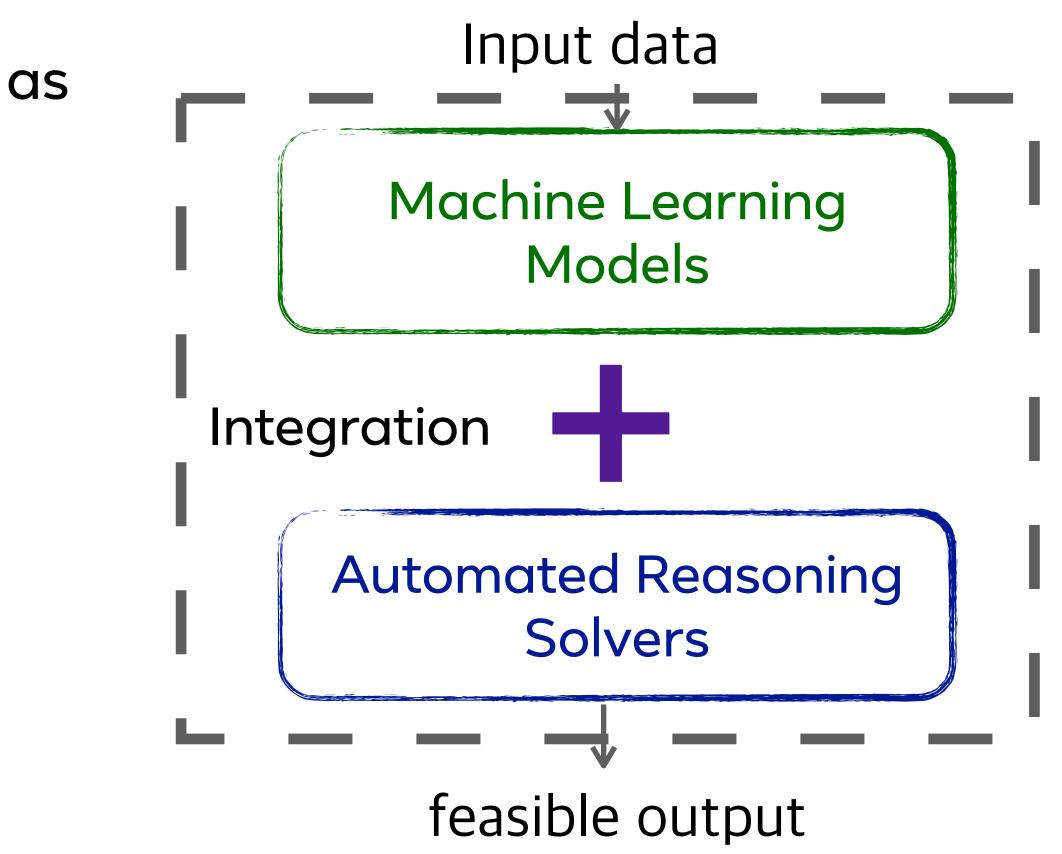




Key insight: Embed reasoning solvers as differentiable modules into neural networks.

The benefits are:

• Formal guarantee of constraint satisfaction.

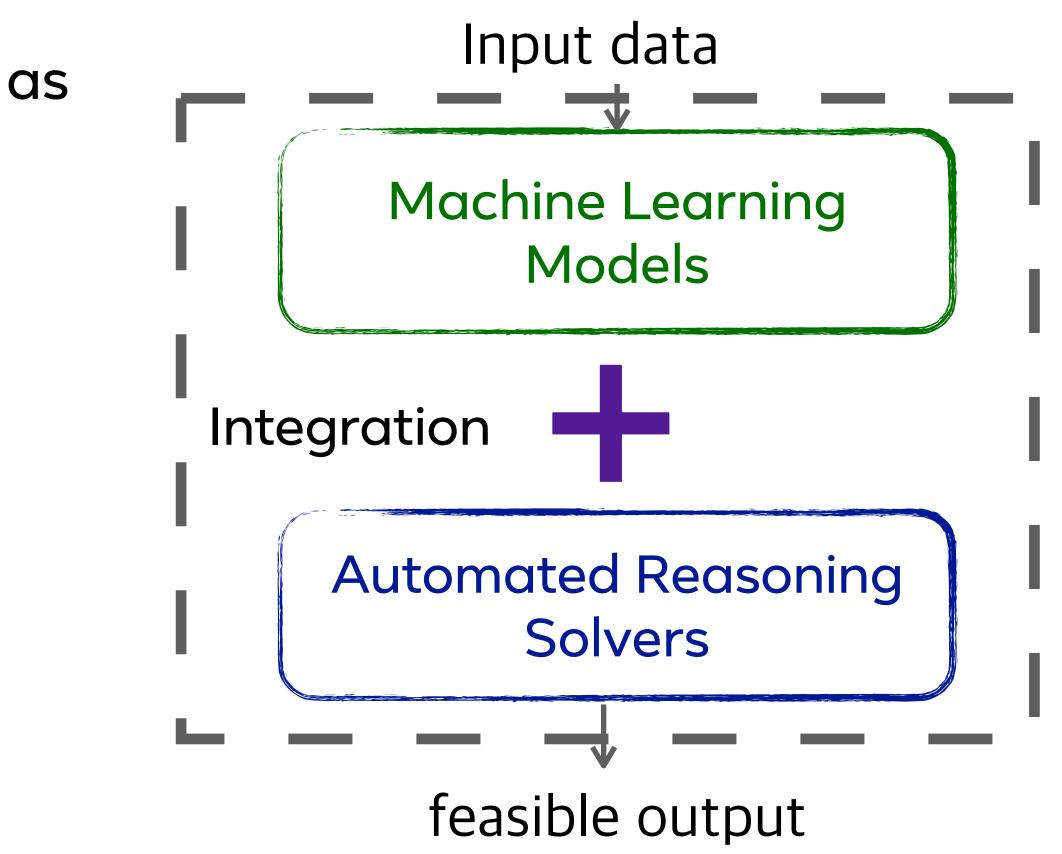




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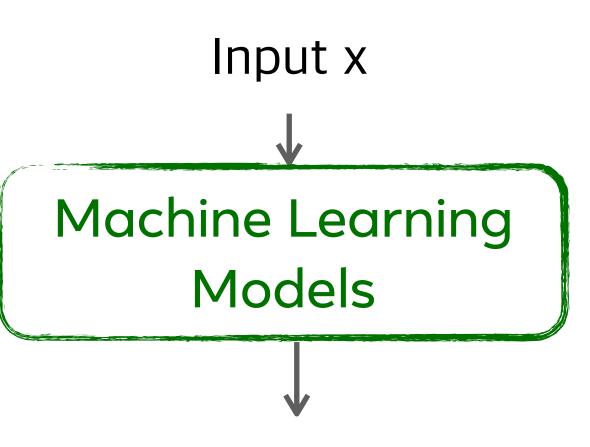
- Formal guarantee of constraint satisfaction.
- Scalability: Accelerate learning for higher-dimensional data.





Design principle of the integrated system

• Learn from data

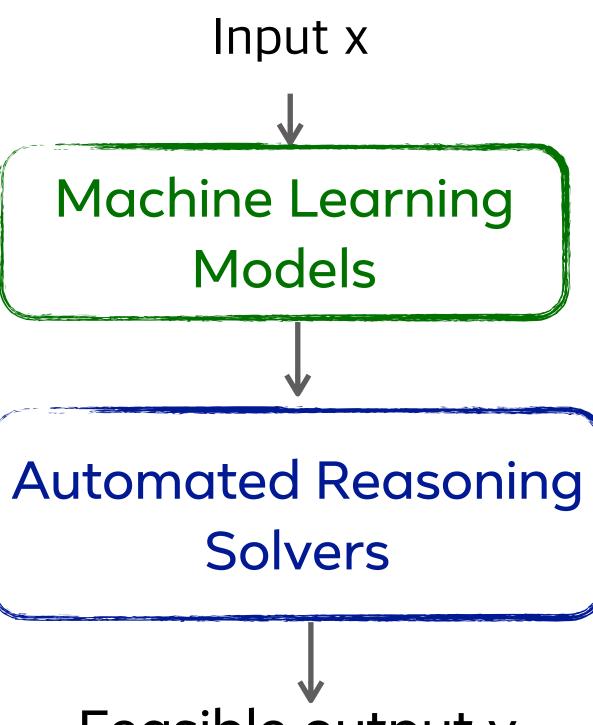




Design principle of the integrated system







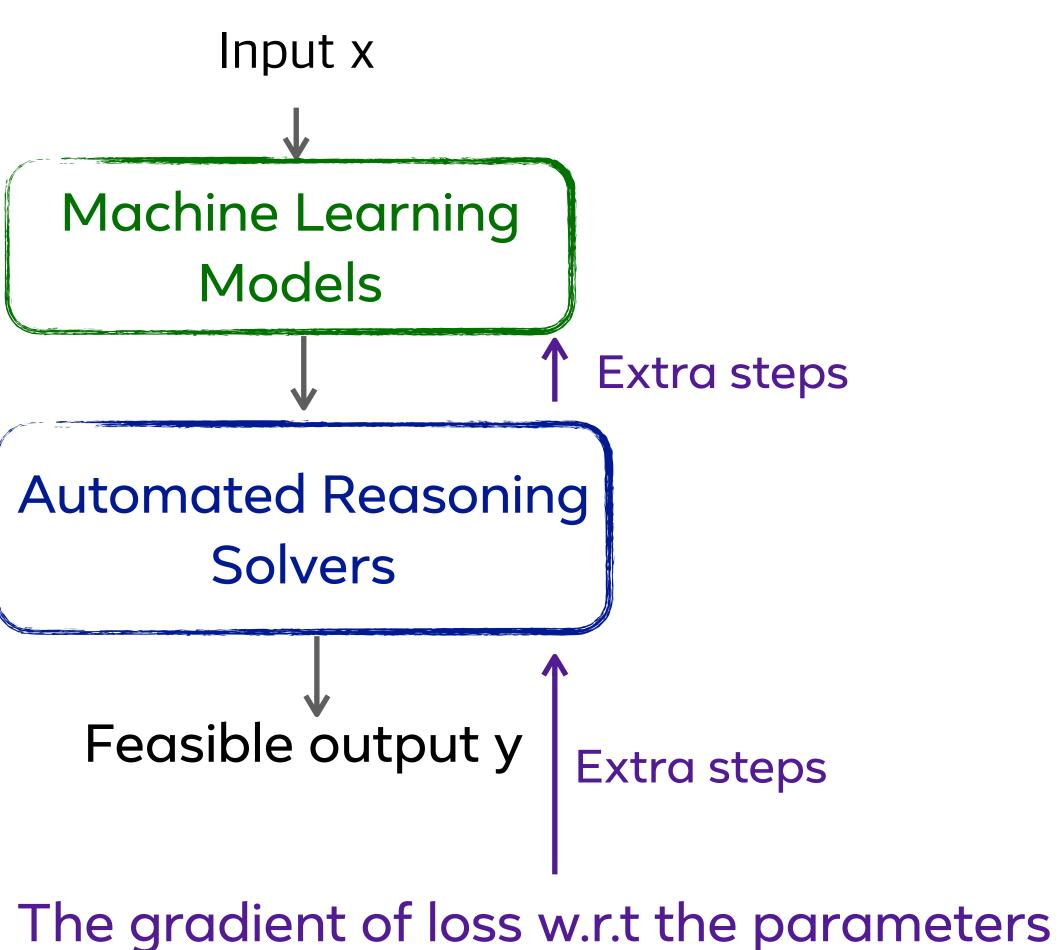
Feasible output y



Design principle of the integrated system

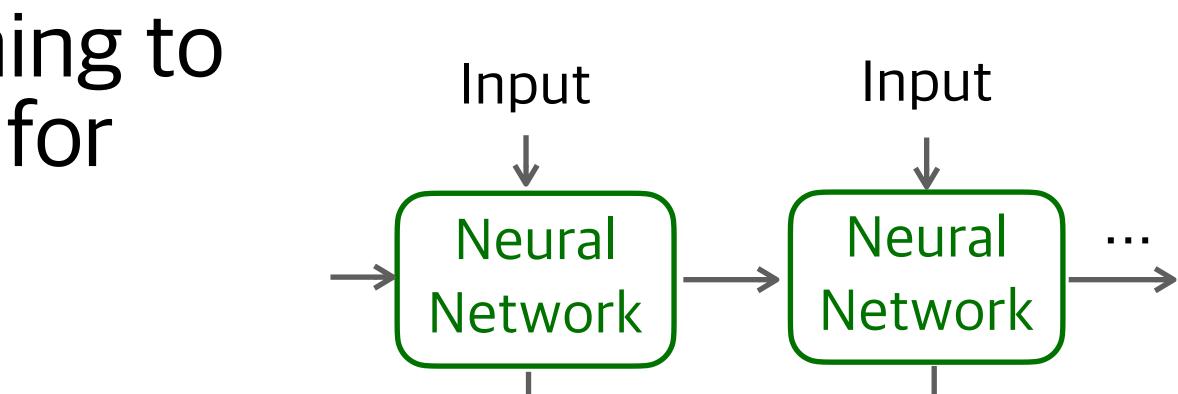




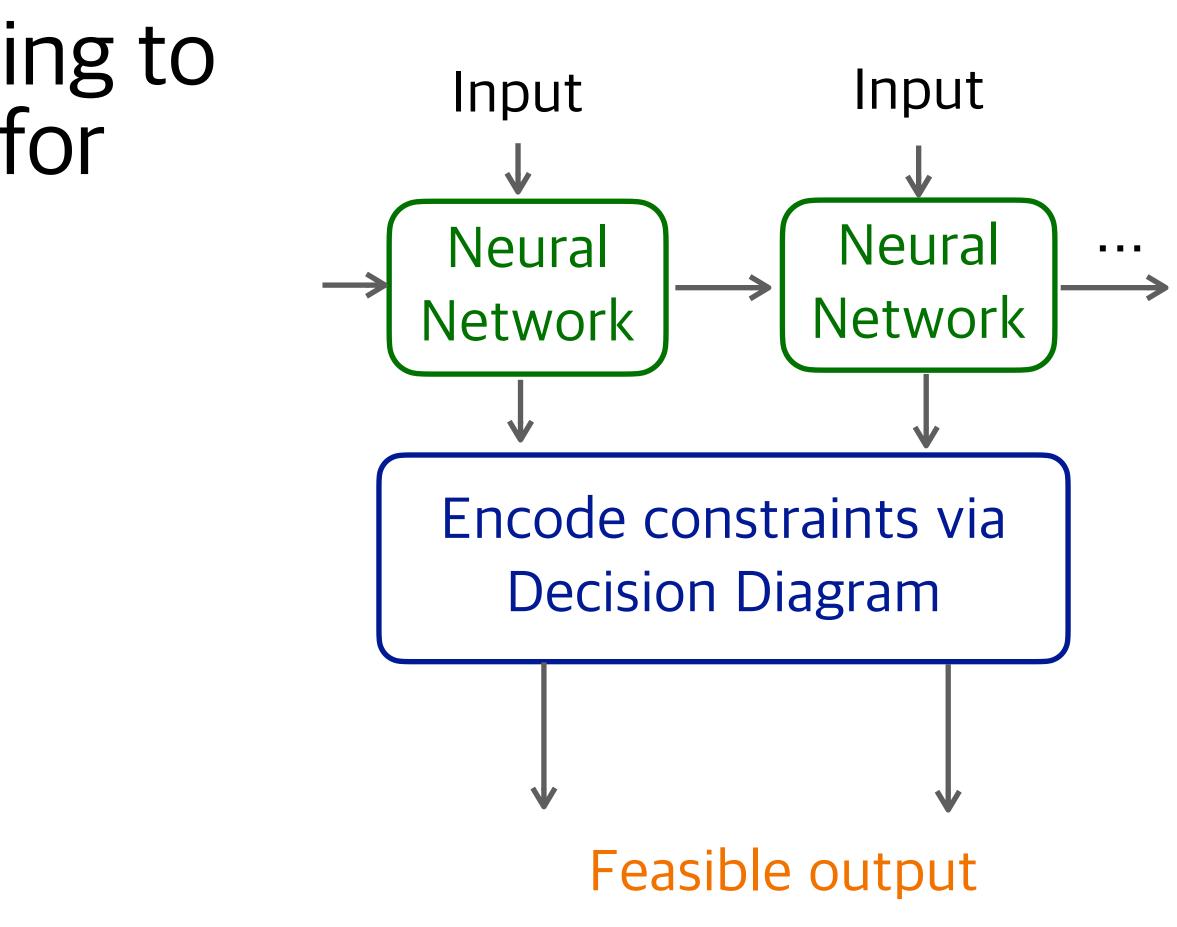


Differentiable

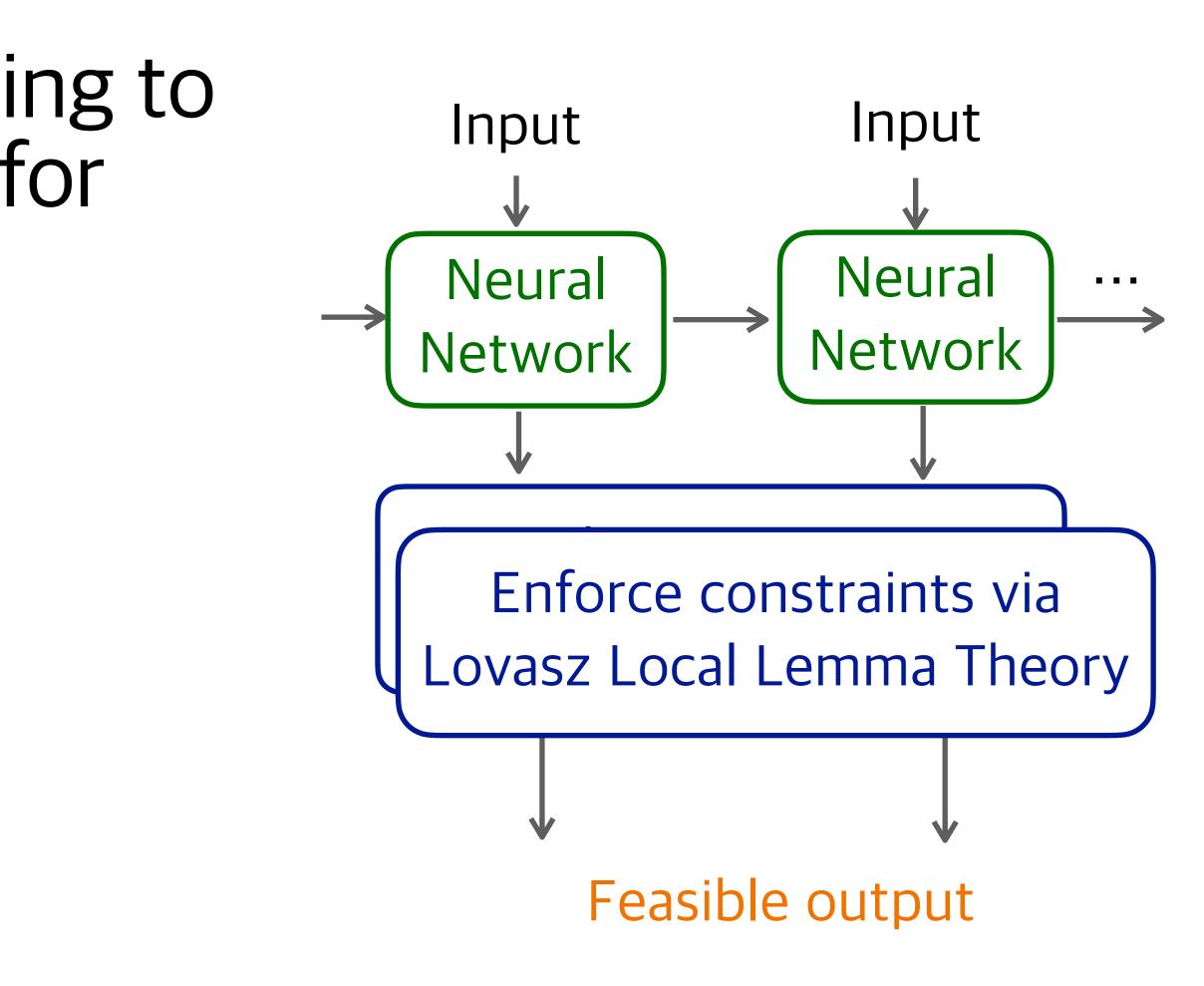




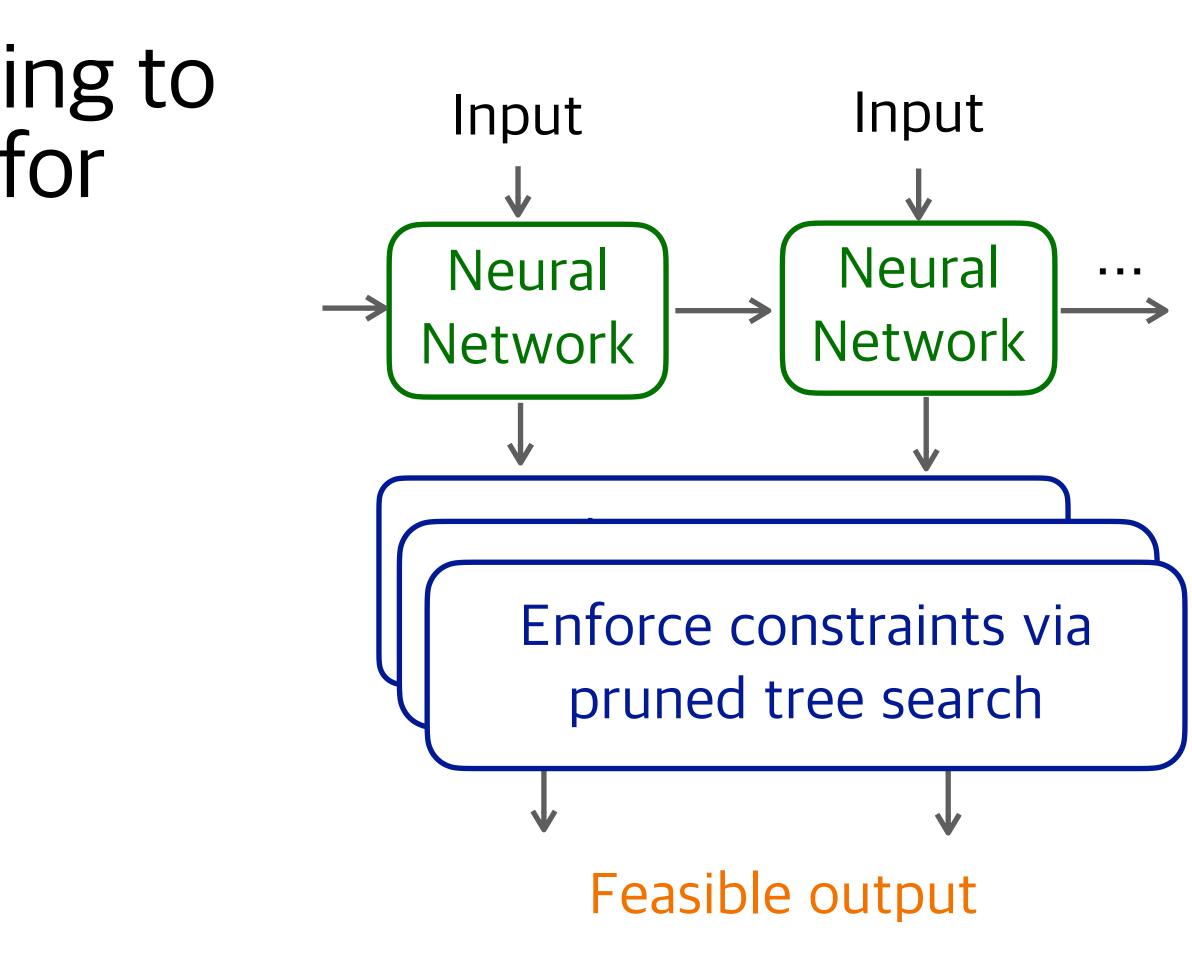






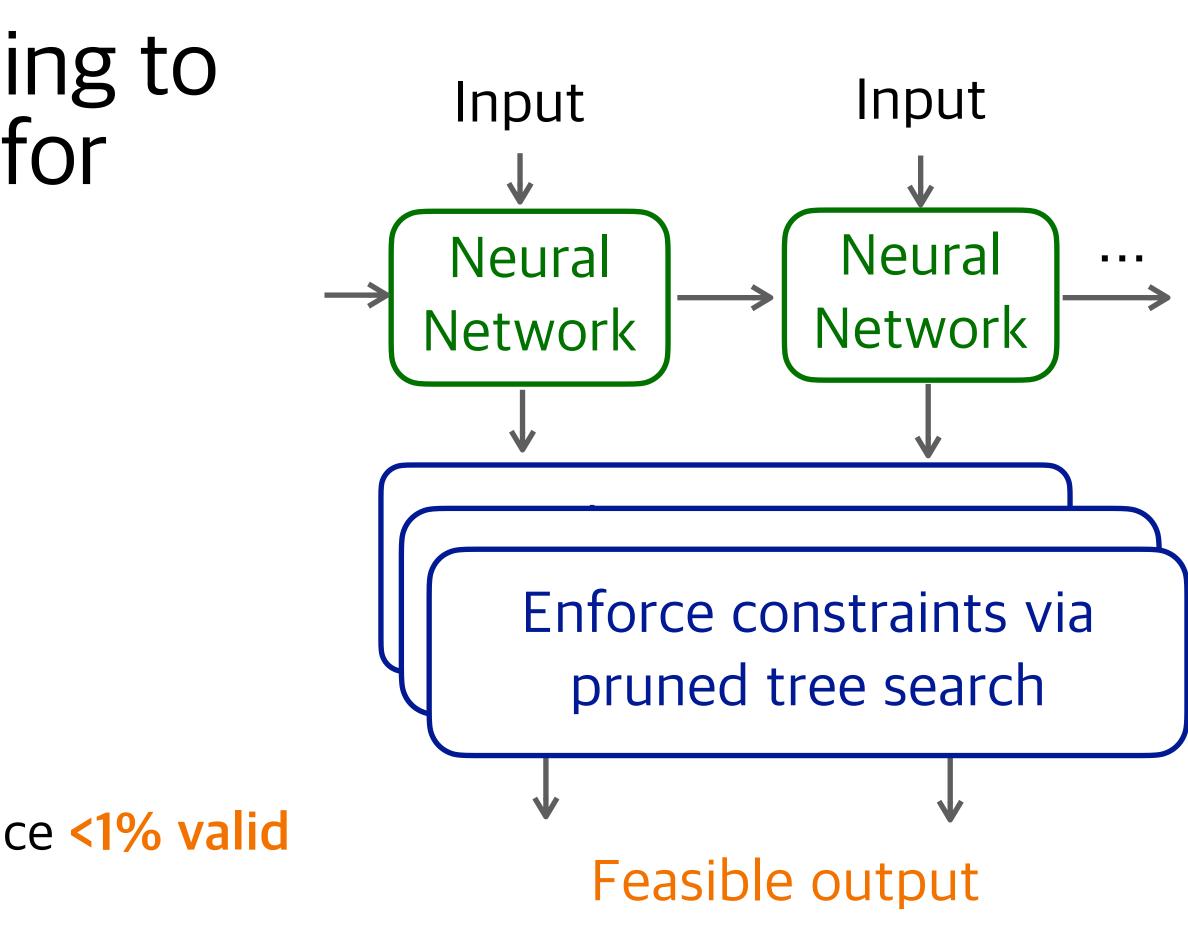








- For route planning,
 - Our method generates 100% valid routes,
 - Pure ML baselines (i.e., Transformer) produce <1% valid routes.





• Task: Learning physical knowledge in closedform from data.

• A series of conference publications: ECML2023, AAAI2024, IJCAI2024, AAAI2025.

Experimental Data

x_1	x_2	x_3	x_4	У
0.2	0.4	0.2	0.7	-0.24
0.9	0.3	0.5	0.5	0.30
0.5	0.4	0.8	0.1	0.36
0.1	0.8	0.7	0.6	-0.41





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- Gap: Pure MLs struggle with <4 variables **setting**, because the search space grows exponentially.
- Our Solution:
 - Combine ML with scientific approach-inspired reasoning.
- Our method discovers scientific equations involving 50 variables.
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Design principle of the integrated system: For CNF-SAT logical constraints satisfying "extreme conditions"

Nan Jiang et al., Learning Markov Random Fields for Combinatorial Structures via Sampling through Lovász Local Lemma. AAAI, 2023.

for CNF-SAT Logical constraint, i.e., $C = \overbrace{(x_1 \lor x_2)}^{c_1} \land \overbrace{(\neg x_1 \lor x_3)}^{c_2}$



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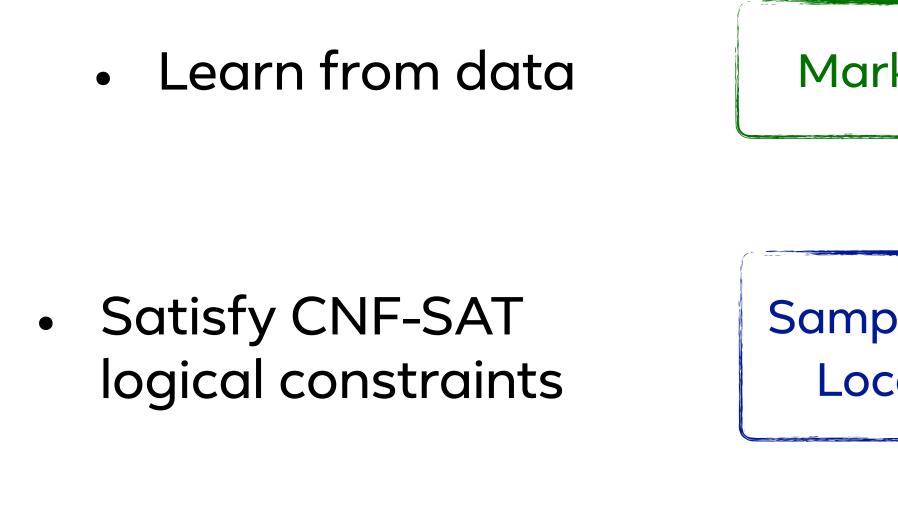
Input x_1, \ldots, x_n

Markov Random Fields

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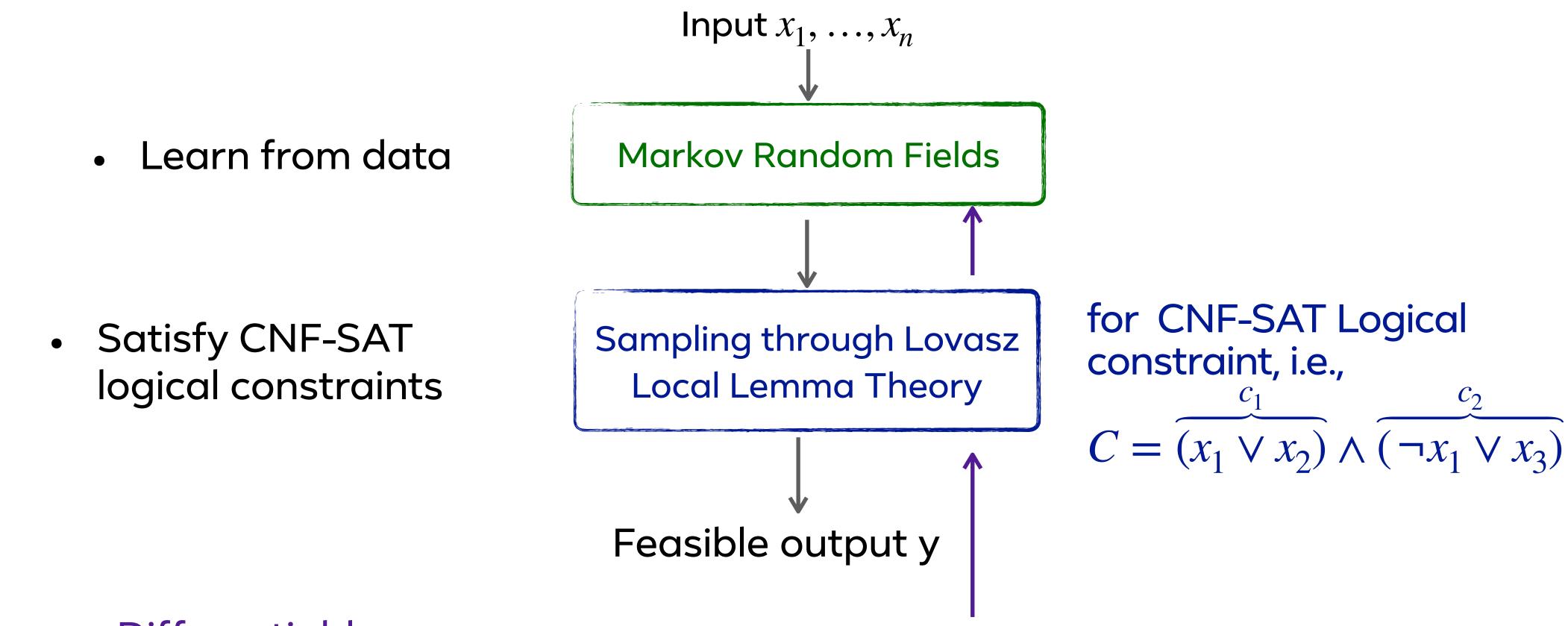
Sampling through Lovasz Local Lemma Theory

for CNF-SAT Logical constraint, i.e., $C = \overbrace{(x_1 \lor x_2)} \land \overbrace{(\neg x_1 \lor x_3)}$

Feasible output y



Design principle of the integrated system: For CNF-SAT logical constraints satisfying "extreme conditions"



Differentiable

Nan Jiang et al., Learning Markov Random Fields for Combinatorial Structures via Sampling through Lovász Local Lemma. AAAI, 2023.

Back-propagate the gradient





• In 1973, Erdos and Lovasz give the existence proof:

there exists a positive probability that none of a series of bad events occur, as long as these events are mostly independent from one another and are not too likely individually.



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In 2010, Moser and Tardos proposed Algorithmic-LLL:

which is an randomized algorithm to find solutions without breaking any bad events

The Gödel Prize 2020 - Laudation

The 2020 Gödel Prize is awarded to Robin A. Moser and Gábor Tardos for their algorithmic version of the Lovász Local Lemma in the paper:

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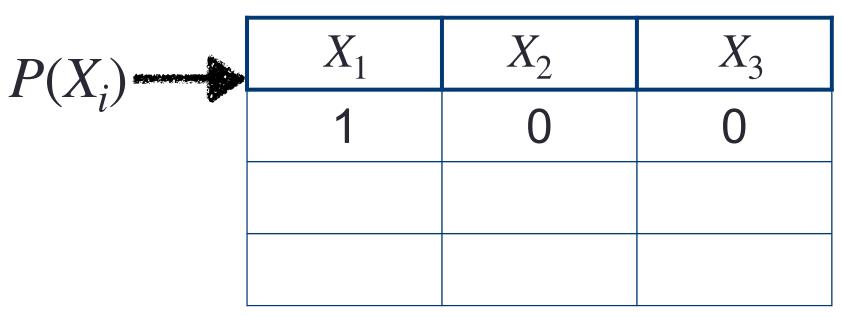
In 2019 (JACM), Heng Guo, Mark Jerrum et al.: The probability distribution of Algorithmic-LLL and the necessary condition



Inputs: Discrete variables $X = [X_1, X_2, X_3]$, with $X_i \in \{0, 1\}$. Marginal distribution: $P(X_1), P(X_2), P(X_3)$; Constraints: $C = \overbrace{(x_1 \lor x_2)}^{c_1} \land \overbrace{(\neg x_1 \lor x_3)}^{c_2}$

Output: A valid sample from distribution $P(x_i | C)$. i=1

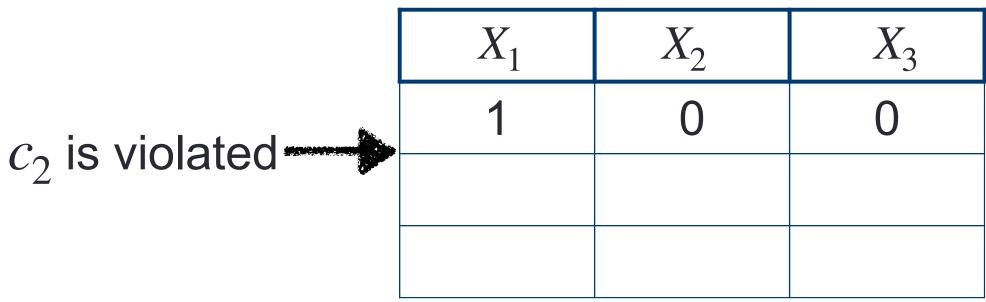
Sample X_i from $P(X_i)$





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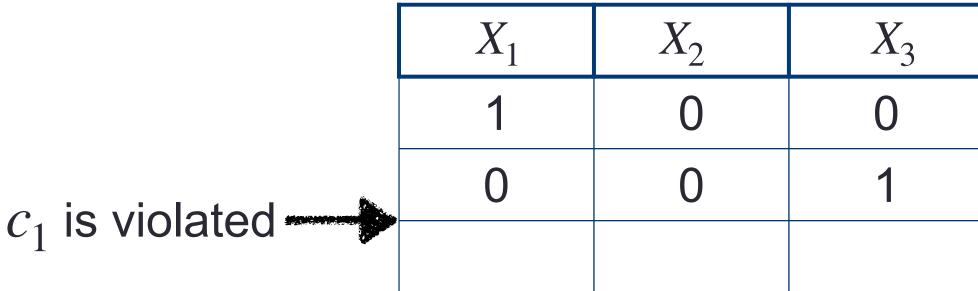
Resample X_1, X_3 from $P(X_1), P(X_3)$

 X_1 X_2 X_3 0 0 0 0



Inputs: Discrete variables $X = [X_1, X_2, X_3]$, with $X_i \in \{0, 1\}$. Marginal distribution: $P(X_1), P(X_2), P(X_3)$; Constraints: $C = \overbrace{(x_1 \lor x_2)}^{c_1} \land \overbrace{(\neg x_1 \lor x_3)}^{c_2}$

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Resample X_1, X_2 from $P(X_1), I$

	X_1	<i>X</i> ₂	<i>X</i> ₃
	1	0	0
$D(\mathbf{V})$	0	0	1
$P(X_2)$	0	1	1



Inputs: Discrete variables $X = [X_1, X_2, X_3]$, with $X_i \in \{0, 1\}$. Marginal distribution: $P(X_1), P(X_2), P(X_3);$ Constraints: $C = \overbrace{(x_1 \lor x_2)}^{c_1} \land \overbrace{(\neg x_1 \lor x_3)}^{c_2}$

Output: A valid sample from distribution $P(x_i | C)$. i=1

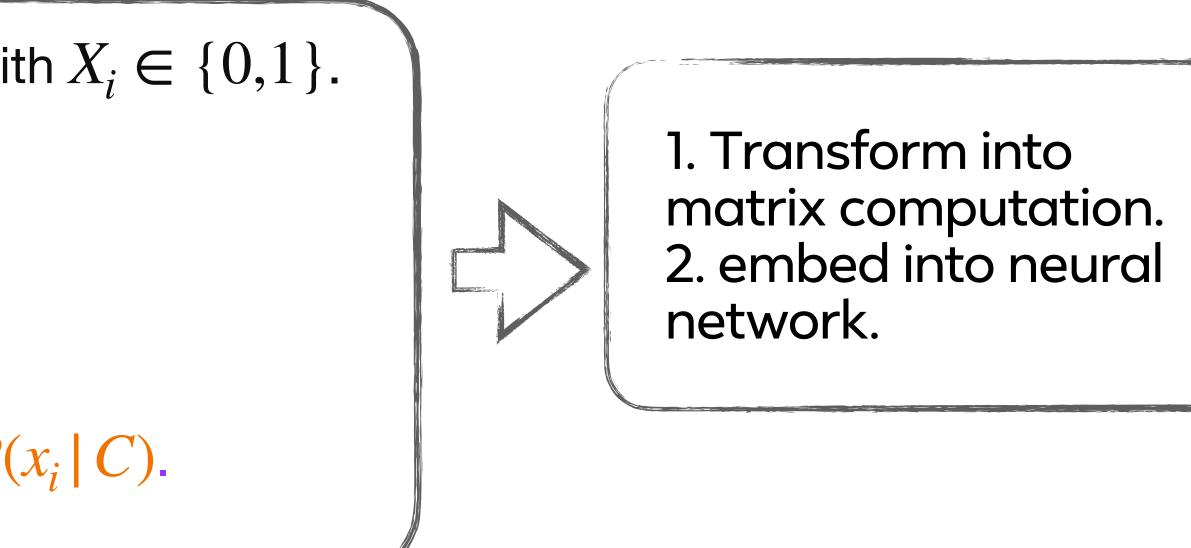
> X_2 X_1 X_3 $\mathbf{0}$ 0 0 0 All constraints are satisfied! 0



Inputs: Discrete variables $X = [X_1, X_2, X_3]$, with $X_i \in \{0, 1\}$. Marginal distribution: $P(X_1), P(X_2), P(X_3)$; Constraints: $C = \overbrace{(x_1 \lor x_2)}^{c_1} \land \overbrace{(\neg x_1 \lor x_3)}^{c_2}$

Output: A valid sample from distribution $\prod_{i=1}^{i=1} P(x_i | C).$

All constraints are sati



	X_1	<i>X</i> ₂	<i>X</i> ₃
	1	0	0
	0	0	1
tisfied!	0	1	1





Implementing Sampling through Lovasz Local Lemma as several Fully Differentiable Neural Network Layers

Input: Discrete variables X_1, X_2, X_3 , with $X_1 \in \{0, 1\}$. Marginal distributions $P(X_1), P(X_2), P(X_3).$ Constraints

$$C = c_1 \wedge c_2,$$

$$c_1 = X_1 \vee X_2,$$

$$c_2 = \neg X_1 \vee X_3$$

$$W = \begin{bmatrix} [1 & 0 & 0] & [0 & 1 & 0] \\ [-1 & 0 & 0] & [0 & 0 & 1] \end{bmatrix}$$

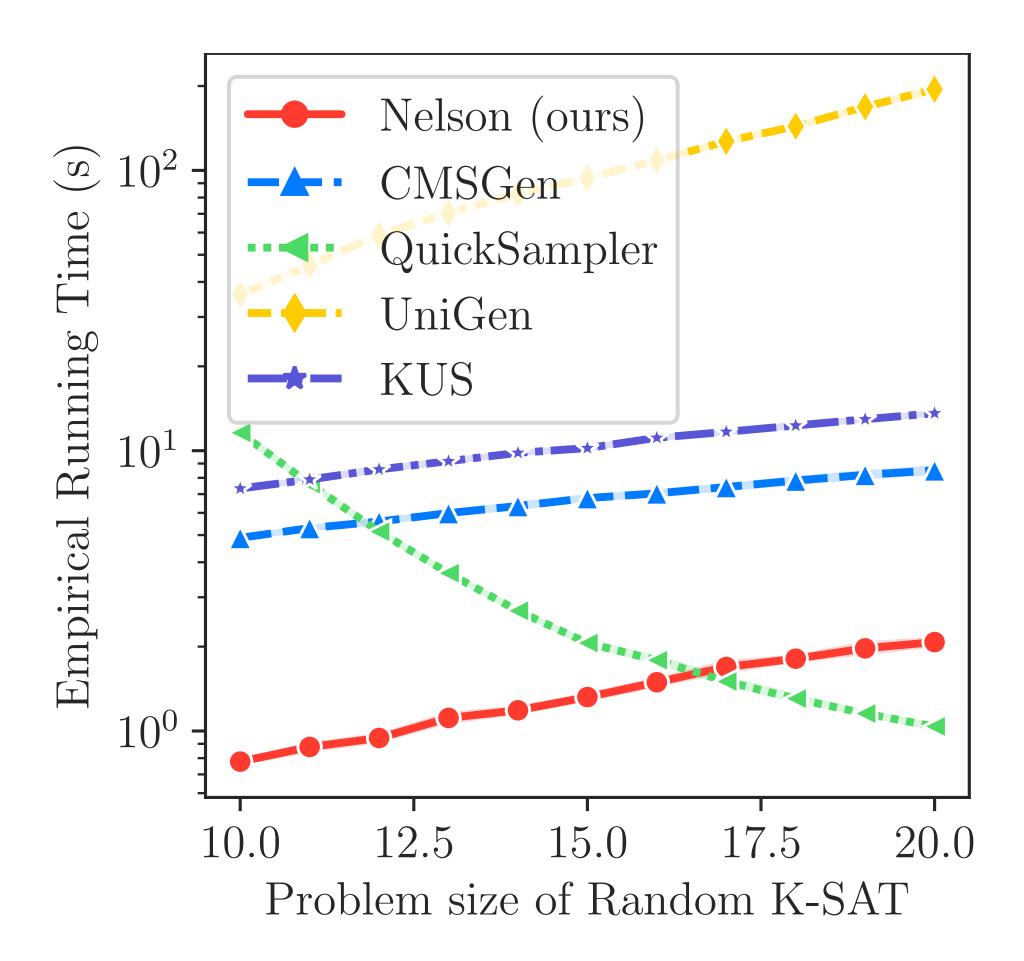
$$b = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

1. Initialize $x_i = 1[u_i > P(X_i)]$ 2. Extract violated constraints $Z = W \otimes x + b$ $S_j = 1 - \max_{1 \le k \le K} Z_{jk}$ 3. Variables to be resamples $A_i = 1 \left| \sum_{i=1}^{L} S_j j V_{ji} \ge 1 \right|$ 4. Resample variables $x = (1 - A) * x + A * 1[u_i > P(X_i)]$ **Fully Differentiable**

Implemented as a series of matrix computations



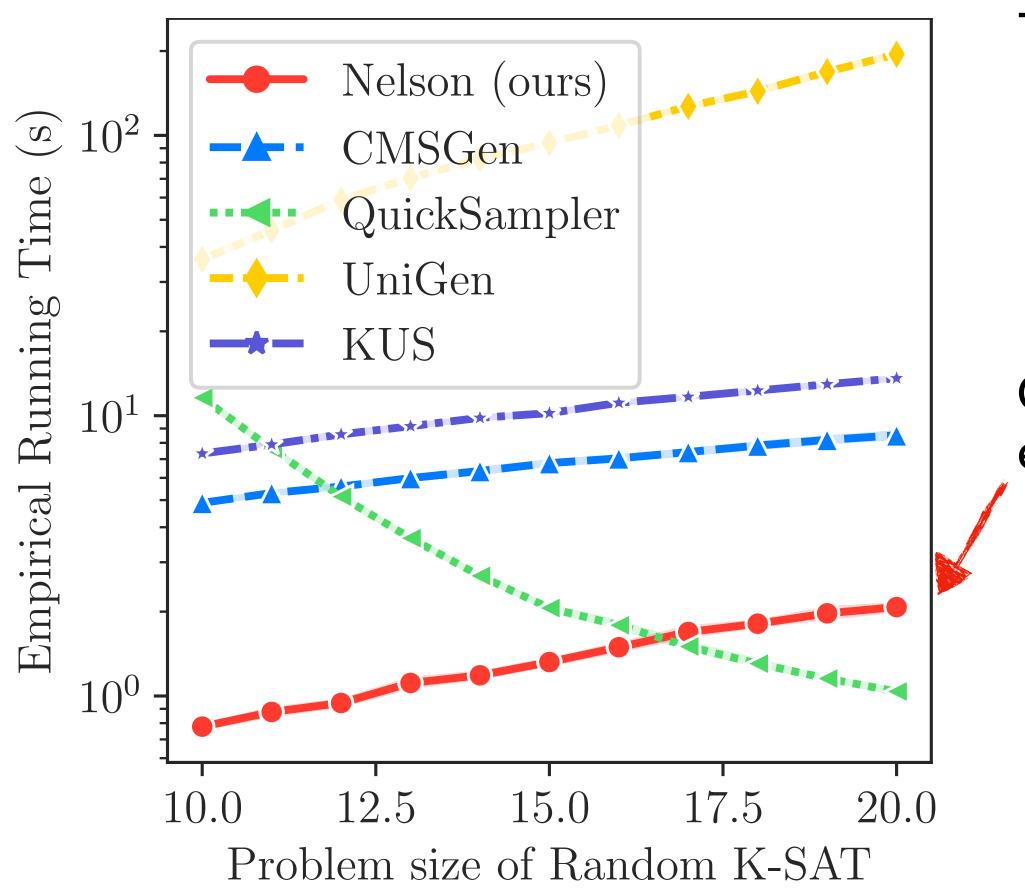


KUS sampler: <u>https://github.com/meelgroup/KUS</u> QuickSampler: https://github.com/RafaelTupynamba/quicksampler/

Experiments: Our Nelson draw samples faster than baselines with constraint satisfaction

Task: sample feasible output from the model.





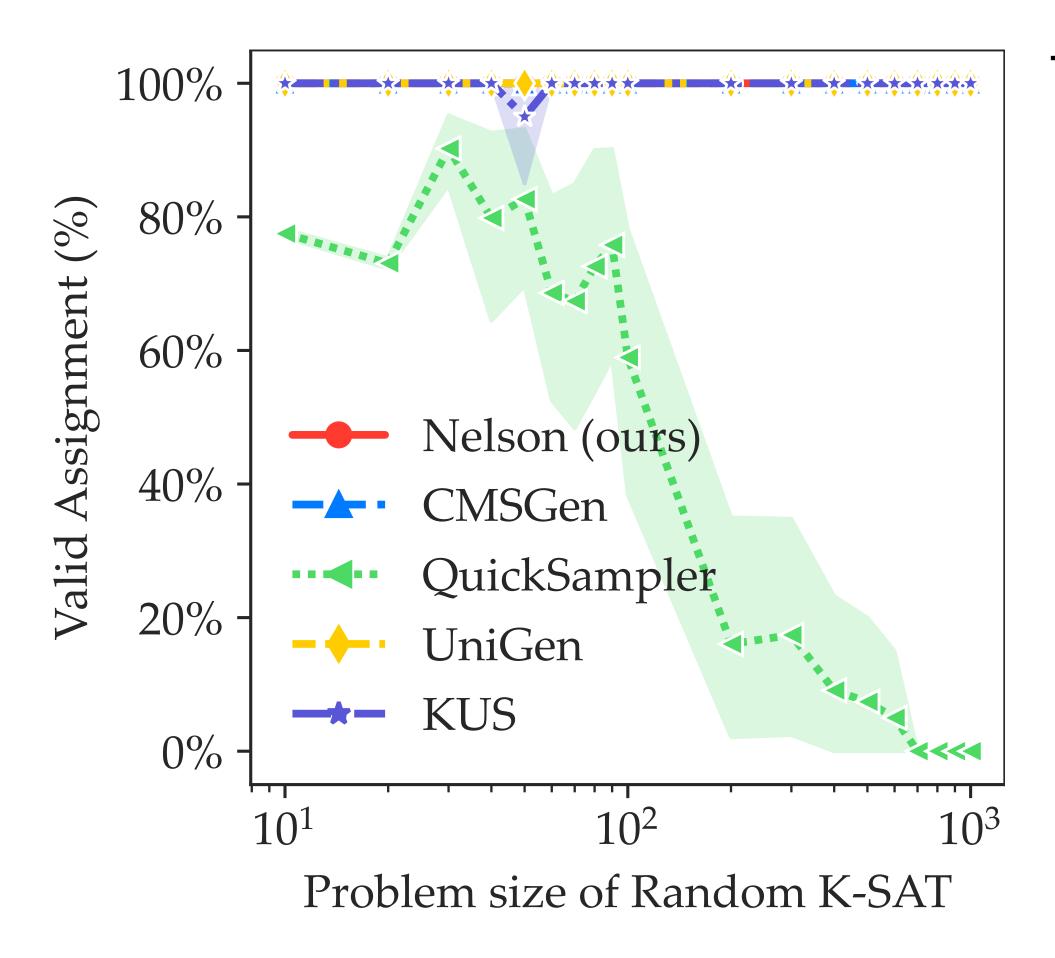
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Our Nelson draw samples faster than existing methods.





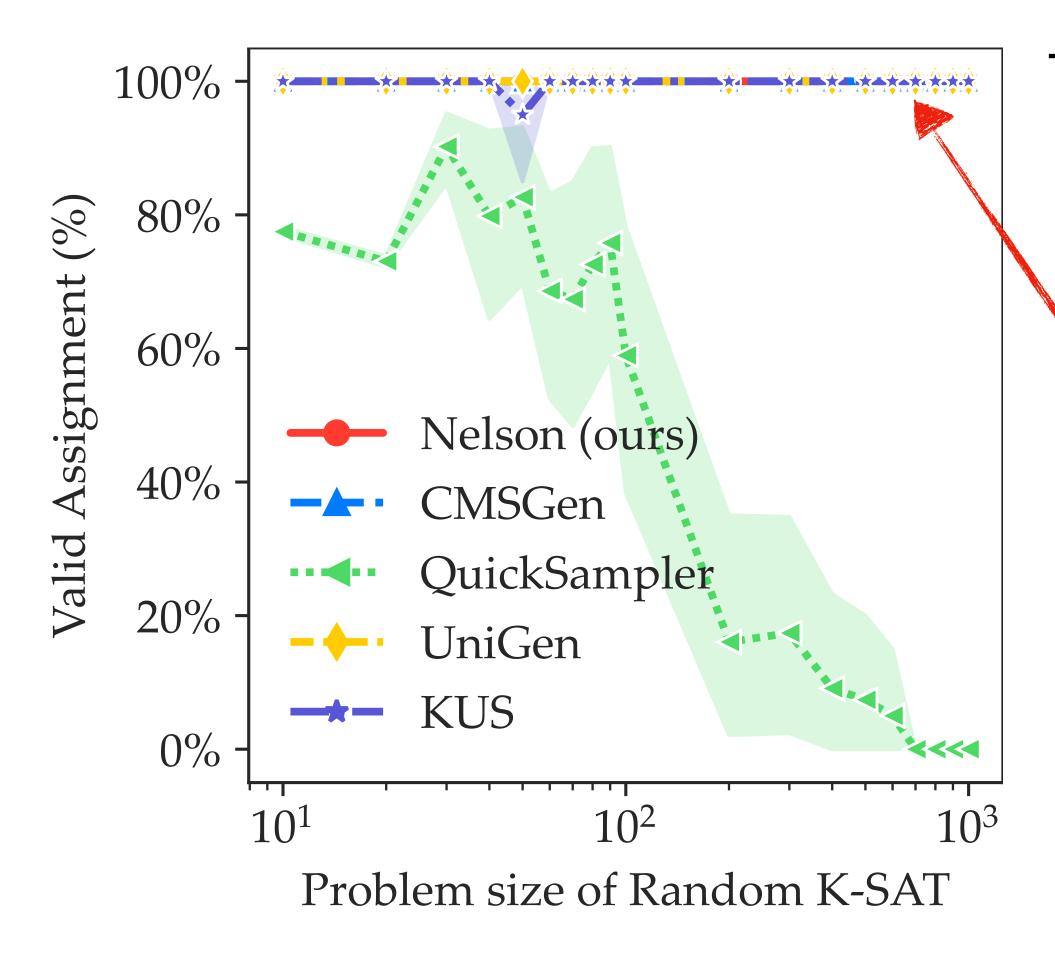
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Task: sample feasible output from the model.

Our Nelson always sample feasible output from the model.

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Experiment: Our Nelson estimates the gradient more accurately

Problem	(a) Training Time Per Epoch (Mins) (\downarrow)									
size	NELSON	XOR	WAPS	WeightGen	CMSGen	KUS	QuickSampler	Unigen	Gibbs	
10	0.13	26.30	1.75	0.64	0.22	0.72	0.40	0.66	0.86	
20	0.15	134.50	3.04	T.O.	0.26	0.90	0.30	2.12	1.72	
30	0.19	1102.95	6.62	T.O.	0.28	2.24	0.32	4.72	2.77	
40	0.23	T.O.	33.70	T.O.	0.31	19.77	0.39	9.38	3.93	
50	0.24	T.O.	909.18	T.O.	0.33	1532.22	0.37	13.29	5.27	
500	5.99	T.O.	T.O.	T.O.	34.17	T.O.	T.O.	T.O.	221.83	
1000	34.01	T.O.	T.O.	T.O.	177.39	T.O.	T.O.	T.O.	854.59	
	(b) Validness of CNF Assignments (%) (↑)									
10 - 50	100	100	100	100	100	100	82.65	100	90.58	
500	100	T.O.	T.O.	T.O.	100	T.O.	7.42	100	54.27	
1000	100	T.O.	T.O.	T.O.	100	T.O.	0.00	100	33.91	
				(c) Approxim	nation Error	of Gradient	(↓)			
10	0.10	0.21	0.12	3.58	3.96	4.08	3.93	4.16	0.69	
12	0.14	0.19	0.16	5.58	5.50	5.49	5.55	5.48	0.75	
14	0.15	0.25	0.19	T.O.	6.55	6.24	7.79	6.34	1.30	
16	0.16	0.25	0.15	T.O.	9.08	9.05	9.35	9.03	1.67	
18	0.18	0.30	0.23	T.O.	10.44	10.30	11.73	10.20	1.90	





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30	0.19	1102.95	6.62	T.O.	0.28	2.24	0.32	4.72	2.77	
40	0.23	T.O.	33.70	T.O.	0.31	19.77	0.39	9.38	3.93	
50	0.24	T.O.	909.18	T.O.	0.33	1532.22	0.37	13.29	5.27	
500	5.99	T.O.	T.O.	T.O.	34.17	T.O.	T.O.	T.O.	221.83	
1000	34.01	T.O.	T.O.	T.O.	177.39	T.O.	T.O.	T.O.	854.59	
	(b) Validness of CNF Assignments (%) (↑)									
10 - 50	100	100	100	100	100	100	82.65	100	90.58	
500	100	T.O.	T.O.	T.O.	100	T.O.	7.42	100	54.27	
1000	100	T.O.	T.O.	T.O.	100	T.O.	0.00	100	33.91	
				(c) Approxin	nation Error of	of Gradient	: (↓)			
10	0.10	0.21	0.12	3.58	3.96	4.08	3.93	4.16	0.69	
12	0.14	0.19	0.16	5.58	5.50	5.49	5.55	5.48	0.75	
14	0.15	0.25	0.19	T.O.	6.55	6.24	7.79	6.34	1.30	
16	0.16	0.25	0.15	T.O.	9.08	9.05	9.35	9.03	1.67	
18	0.18	.30	0.23	T.O.	10.44	10.30	11.73	10.20	1.90	

Our Nelson estimates the gradient more accurately

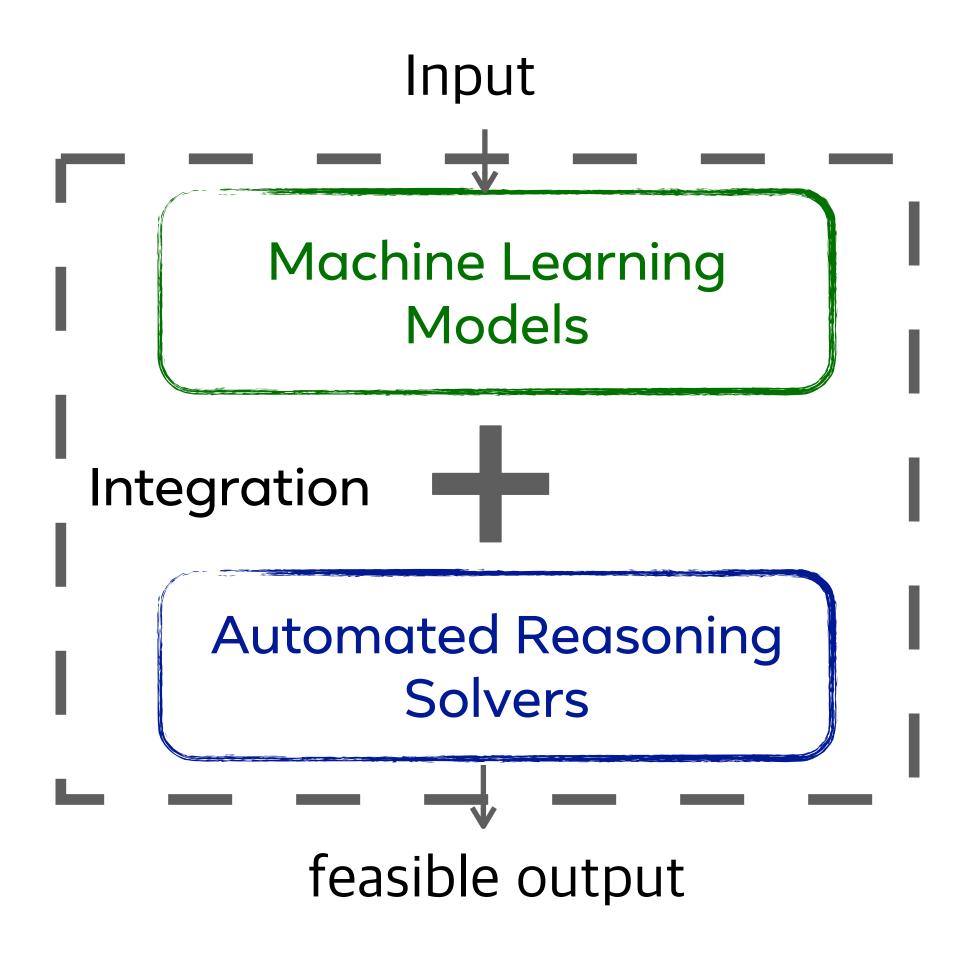




Takeaway

The benefits are:

- Formal guarantee on Constraint satisfaction.
- Scalablilty: Accelerate learning for higher-dimensional data.





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